Harmonic Reduction and Elimination in Three Phase PWM Inverters using a Spiral-Inspired Optimization Technique

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ABSTRACT/RESUME

Abstract: The quality of the inverter output is a major concern when renewable energies are to be connected to the grid. Particularly, the harmonic distortion is a headache to the system designer and may lead to malfunctioning of the overall system. In this work, the problem of harmonic elimination through optimizing the firing angles in a three phase PWM inverter is addressed. Harmonic elimination techniques give an improved performance by cancelling the most problematic harmonics. The Spiral optimization technique results in a further reduction in the harmonic distortion. In this approach, the ONand OFF instances (firing angles) of the switches to eliminate some desired harmonics are pre-calculated. These firing angles are stored in a microprocessor which produces the pulses with predetermined timing corresponding to the desired harmonics to be eliminated.

I. Introduction

Over the last years, there have been major advancements in power electronics. Power electronics has moved on based on these developments with such things as digital signal processors being used to control power systems. An Inverter is basically a converter that converts DC to AC power. Inverter circuits can be very complex. A voltage source inverter (VSI) is one that takes in a fixed voltage from a device, such as a dc power supply or a PV solar energy panel, and converts it to a variable-frequency AC supply. Voltage-source inverters are divided into three main categories: Pulse-width Modulated (PWM) Inverters, Square-wave Inverters and Single-phase Inverters with Voltage Cancellation. Pulse-width modulation inverters take in a constant DC voltage. Diode-rectifiers are used to rectify the line voltage, and the inverter must control the magnitude and the frequency of the AC output voltages. To do this, the inverter uses pulse-width modulation using its switches. There are different methods for implementing the pulse-width modulation in an inverter in order to shape the output AC voltage to be very close to a sine wave. These different methods will be detailed further with a focus on sinusoidal-PWM. Square-wave inverters have their input connected to a controlled DC voltage in order to control the magnitude of the output AC voltage. The inverter controls only the frequency of the output while the input voltage controls the magnitude. The output AC voltage has a waveform similar to a square wave, based on which the inverter got its name. Single-phase inverters with voltage cancellation take in a constant DC source and output a square-wave like AC voltage. They can control both frequency and magnitude of the output but do not use PWM and therefore have a square-wave like output. These inverters have joint characteristics of the previous two inverters. The voltage cancellation only works with single phase inverters. Three-phase controlled converters have many applications such as AC and DC adjustable speed drives (ASD) [1]-[5], induction heating, HVDC power systems, power supplies and interfacing of renewable energy (RE) systems with electric utilities [6]-[10]. These applications use controlled converters such as a rectifier or an inverter. The line currents of controlled converters have high harmonic content with respect to the PWM converters that use IGBT. However, apart from the
II. Harmonic Reduction in Inverters

II.1. DC/AC Inverters

DC to AC inverters are those devices which are used to perform inversion by converting a direct current into an alternating current. If the output of a circuit is AC then depending on the input i.e. either

converters in several applications. The heart of the work in the SHE-PWM techniques is to be able to get the analytical solution to the system of nonlinear transcendental equations containing trigonometric terms and that turns out to have multiple sets of solutions [34],[41],[42],[43]–[44]. Several algorithms have been reported in the literature regarding methods of solving the resultant set of nonlinear transcendental equations which describe the SHE-PWM problem [45–46]. These can be categorized into three classes: analytical approach on resultant theory method [47], numerical iterative techniques, such as Newton–Raphson method [48] and evolutionary algorithms [49–50] such as genetic algorithm (GA) or particle swarm optimization (PSO) etc. The Newton–Raphson iterative approach [33], [51], [35] is derivative-dependent and may get trapped at local optima. Furthermore, a careful choice of the initial guess must be done to guarantee the convergence of the algorithm [52], [44]. A useful harmonic elimination scheme for multilevel converters was reported in [53]. The method reduces the number of the equations defining the harmonic elimination into four simple equations with minimum calculation time. The equal area criterion is used to obtain the solution to the angles through a simple iteration procedure. However, the performance of this method is related to the number of switching angles and the number of selected harmonics. As a result, applying directly this method would not guarantee finding best switching angles for all the modulation indexes and voltage step combinations. There have been several optimization algorithms used to address the SHE problem: genetic algorithms [54], particle swarm optimization [55], bacterial foraging [56], ant colony [57] and ABC algorithm [58]. Genetic algorithms have been introduced to optimize the sequence of the carrier waveform of the PWM so as to minimize the THD and the distortion factor (DF) of output line voltage [59]. An optimization technique assisted with a hybrid genetic algorithm was successfully applied to find the switching transitions (i.e., switching angles) of the SHE-PWM AC/AC converter [60]. The main purpose of this paper is to design converters which produce desired output with a fewer harmonics. This is done by suitable control strategies and optimization techniques to achieve harmonics-free output in multilevel inverters. Mainly, the spiral technique is involved in the optimization task.

II. Harmonic Reduction in Inverters

II.1. DC/AC Inverters

DC to AC inverters are those devices which are used to perform inversion by converting a direct current into an alternating current. If the output of a circuit is AC then depending on the input i.e. either

higher switching losses associated with PWM converters, the power handling capability and reliability of these devices are quite low when compared to the Silicon Controlled Rectifiers (SCRs) [10]. Moreover, some applications, especially in high ratings, favor line commutated converters over PWM due to the high Electromagnetic Interference associated with PWM. Many techniques have been introduced to reduce these harmonics such as a reduction by using increased pulse numbers, IPN [11], active and passive filters, APF [12]–[14], modulation of the controlled signal of DC-DC converters connected to a converter by third harmonic components, MCC [15], or by third harmonic injection from the DC-link to the line currents, 3rd_INJ [16]–[19]. The third harmonic injection technique was used in uncontrolled converters in [20]–[23]. A review of the three-phase improved power quality of uncontrolled converters by different techniques is shown in [24]. The third harmonic injection technique in a controlled converter was first introduced in [25]. This technique uses the third harmonic voltage in the DC-link to inject a current to the line currents. References [26] and [27] used the three LC branches tuned around triple the utility frequency to inject the third harmonic current into the line currents. This technique has many disadvantages due to its high cost, its bulkiness, and its need of precise values for L and C to divide the third harmonic currents equally. References [28], [29] used an interfacing delta-star transformer to circulate the injection current to the neutral of the star. This technique increases the cost due to the interfacing transformer. Other references [18], [19] used star-delta transformer in the re-injection path with an unloaded delta to circulate the injection current through the neutral of the star to the line currents. Some other references [16], [20], [30], and [31] used a partial rating (20%) zigzag transformer to circulate the third harmonic injection current to the line currents to replace the need for a full load Δ/Y transformer. The injection of the third harmonic has been controlled by using a single-phase boost rectifier to circulate the power in the third harmonic path back to the DC-link to increase the converter efficiency [31]. This technique is suitable for uncontrolled rectifiers because it provides an easy way to control the third harmonic current in the injection path, but it cannot control the angle of the injection current that should be varied with changing the firing angle of the single-phase controlled converter [31], [32].

Selective harmonic elimination based on pulse-width modulation (SHE-PWM) has been developed for two and three-level converters in order to have lower total harmonic distortion (THD) in the voltage output waveform [33–35]. It has been then extended to diverse multilevel [36–37] and hybrid multilevel [38–40] converters in several applications. The heart of the work in the SHE-PWM techniques is to be able to get the analytical solution to the system of nonlinear transcendental equations containing trigonometric terms and that turns out to have multiple sets of solutions [34],[41],[42],[43]–[44]. Several algorithms have been reported in the literature regarding methods of solving the resultant set of nonlinear transcendental equations which describe the SHE-PWM problem [45–46]. These can be categorized into three classes: analytical approach on resultant theory method [47], numerical iterative techniques, such as Newton–Raphson method [48] and evolutionary algorithms [49–50] such as genetic algorithm (GA) or particle swarm optimization (PSO) etc. The Newton–Raphson iterative approach [33], [51], [35] is derivative-dependent and may get trapped at local optima. Furthermore, a careful choice of the initial guess must be done to guarantee the convergence of the algorithm [52], [44]. A useful harmonic elimination scheme for multilevel converters was reported in [53]. The method reduces the number of the equations defining the harmonic elimination into four simple equations with minimum calculation time. The equal area criterion is used to obtain the solution to the angles through a simple iteration procedure. However, the performance of this method is related to the number of switching angles and the number of selected harmonics. As a result, applying directly this method would not guarantee finding best switching angles for all the modulation indexes and voltage step combinations. There have been several optimization algorithms used to address the SHE problem: genetic algorithms [54], particle swarm optimization [55], bacterial foraging [56], ant colony [57] and ABC algorithm [58]. Genetic algorithms have been introduced to optimize the sequence of the carrier waveform of the PWM so as to minimize the THD and the distortion factor (DF) of output line voltage [59]. An optimization technique assisted with a hybrid genetic algorithm was successfully applied to find the switching transitions (i.e., switching angles) of the SHE-PWM AC/AC converter [60]. The main purpose of this paper is to design converters which produce desired output with a fewer harmonics. This is done by suitable control strategies and optimization techniques to achieve harmonics-free output in multilevel inverters. Mainly, the spiral technique is involved in the optimization task.
AC or DC, the devices are called as AC-AC cycloconverters or DC-AC inverters, respectively. DC to AC inverters are devices whose AC output has magnitude and frequency which is either fixed or variable. In the case of DC to AC inverters, the output AC voltage can be either single phase or three phase. Also, the magnitude of the AC voltage is in the range of 110-380 VAC while the frequencies are either 50Hz, 60Hz or 400Hz. Some of the basic applications of inverters would be an UPS (uninterruptible power supply). When the main power is not available UPS uses batteries and inverter to supply AC power. A rectifier is used to recharge the batteries used when the main power is back. Other applications of an inverter included Variable frequency drives. The variable frequency drives controls the frequency and voltage of power supplied to the motor, thus controlling the speed of AC motor. An inverter is used in the variable frequency drives to provide controller power. An inverter is also used in an induction motor to regulate the speed by changing the frequency of AC output.

II.2. Three Phase Inverters
Similar to the Single Phase Inverters, the Three Phase Inverters also have different topologies which can be used. Figure 1 shows a three phase inverter circuit. It is an extension of H-bridge circuit as it consists of three single phase inverters each connected to one of the three load terminals. In the case of single phase inverter, there is a phase shift of 180 degrees between different legs, while in case of three phase inverter there is a phase shift of 120 degrees. This phase shift of 120 degrees in three phase inverter helps in eliminating the odd harmonics from the three legs of the inverter. Also, if the output is pure AC waveform then the even harmonics can be eliminated as well. In order to modulate the output of a three phase inverter, the amplitude of output voltage is reduced by a factor with respect to the input voltage.

II.3. Multilevel Inverters
Multi level Inverters are a type of inverters whose construction is similar to the single and three phase inverters as explained earlier. Figure 2 shows a multi level inverter which is an extension of single and three phase inverters. Here, four IGBT circuits are connected in three different legs and the diodes are connected in parallel to each leg in opposite direction. Also, the loads are connected between two IGBT circuits for each leg as shown in Fig. 2. The advantages of using multi level inverters instead of single and three phase inverters are namely:

- Multi level inverters can be used for higher voltage levels
- Multi level inverters have higher capability of reducing the harmonics because of multiple DC levels. The term “multilevel” finds its origin from the three-level converters [61]. The idea is that by increasing the number of levels in a specified configuration, the output voltages would generate a staircase waveform with more steps which makes it approach roughly the desired sinusoidal waveform and also exhibits a reduced harmonic distortion. The more levels are added, the closer the output to approximate the sinewave and the lower is the system’s THD [61-65].

II.4. Methods for Harmonic Reduction in Inverters
One of the most important aspects of a system is the reduction of harmonics that are present in the system. In case of an inverter, it is very important to remove the harmonics from the AC output.
The harmonics present in a DC to AC inverter are very much obvious compared to the harmonics that can be present in an AC to DC converter. This is because of the output of DC to AC inverter is AC. Thus, the filters that are used in DC to AC inverters have different designs compared to the filters used in AC to DC converters. In case of AC to DC converters, the main objective is to improve the output voltage ripple. Thus, passive filters can be easily used in order to improve the output of an AC to DC converter. While, in case of DC to AC inverter, the harmonic reduction is harder and it thus involves the use of active filters.

As the output of the DC to AC inverters is alternating, it is very important to produce pure sinusoidal output waveforms. In order to produce such sinusoidal waveforms, filters are implemented which reduce the harmonic effect by removing the third and higher harmonics from the system. The filters used to remove the harmonics from the inverters are more complex and consist of a large number of inductors and capacitors to remove the harmonics of higher order. This also results into more costly filters to remove harmonics from the inverter. Thus, in order to avoid the cost of such expensive and complex filters, controlling the width or reducing the number of pulses may result in a reduction of harmonics. One such technique is the PWM technique explained below.

II.5. Pulse Width Modulation Technique

In a single phase inverter, varying the width of the output pulse is used to control the output voltage. This process of controlling the output voltage of inverter in order to reduce the harmonics is known as Pulse Width Modulation (PWM). The Pulse Width Modulation is classified into two techniques:

- Non sinusoidal Pulse Width Modulation
- Sinusoidal Pulse Width Modulation

a- Non Sinusoidal Pulse Width Modulation

In the case of Non sinusoidal pulse width modulation, all the pulses that have the same pulse width are modulated together. The pulse widths are adjusted together in same proportion on order to remove the harmonics from the system. A typical representation of Non sinusoidal pulse width modulation is shown in figure 3.

b- Sinusoidal Pulse Width Modulation

Sinusoidal Pulse Width Modulation is different as compared to the nonsinusoidal Pulse Width Modulation. In the case of sinusoidal pulse width modulation, all the pulses are modulated individually. Each and every pulse is compared to a reference sinusoidal signal and then they are modulated accordingly to produce a waveform which is equal to the reference sinusoidal waveform. Figure 4 shows a representation of Sinusoidal Pulse Width Modulation.

III. The Spiral Inspired Optimization Method

In the present paper, the use of the spiral optimization technique is presented and used to solve the switching angles in PWM inverters. The purpose is to reduce the total harmonic distortion factor at the output to satisfy the standards regarding the harmonic content of grid-connected renewable energy system. A high THD value can:

1. cause an excess in the current draw from the power systems which results in higher temperatures;
2. shorten the life of electronic equipment and, (iii) reduce the Power Quality. This comprises also the electronic systems that are very sensitive such as computers, communication systems and controllers that may respond incorrectly to inputs affected by harmonics. For that, the study of power quality and mainly harmonics elimination or THD minimization in power systems is becoming a major concern of power system engineers [66-67].
difference between local and global search of optimization techniques is that the local techniques produce results that depend highly on the starting point or the initial guess, while the global methods are independent of the initial conditions, despite the fact that they are fast in convergence, the local techniques have a direct dependence on the existence of at least the first derivative. Furthermore, they place constraints on the solution space such as differentiability and continuity. These conditions are hard or even impossible to satisfy in practice. The global techniques, on the other hand, place fewer constraints on the solution space.

Compared with traditional optimization techniques and other global optimizers, the spiral optimization method turns out to be easy to implement and very efficient in reaching optimum solutions. Spiral optimization method has been recently developed based the analogy to spiral phenomena [67].

Spiral Dynamics is a theory of human development. Spiral Dynamics argues that human nature is not fixed: humans are able, when forced by life conditions, to adapt to their environment by constructing new, more complex, conceptual models of the world that allow them to handle the new problems [67]. Each new model transcends and includes all previous models. Within the model, individuals and cultures do not fall clearly in any single category (color). Each person/culture embodies a mixture of the value patterns, with varying degrees of intensity in each. Spiral Dynamics claims not to be a linear or hierarchical model although this assertion has been contested.

The colors act as reminders for the life conditions and alternate between cool and warm colors as a part of the model [68].

According to Spiral Dynamics, there are infinite stages of progress and regression over time, dependent upon the life circumstances of the person or culture, which are constantly in flux. Attaining higher stages of development is not synonymous with attaining a "better" or "more correct" values system. All stages co-exist in both healthy and unhealthy states, meaning any stage of development can lead to undesirable outcomes with respect to the health of the human and social environment [67].

The spiral phenomena occurring in nature are approximated by logarithmic spirals as shown in Fig. 5. Examples of natural spiral dynamics include whirling currents, low pressure fonts, nautilus shells and arms of spiral galaxies. Logarithmic spiral discrete processes generate spirals that can form an effective behaviour in metaheuristic. A two-dimensional algorithm has been first proposed [67], and then, a more generalized n-dimensional version has been recently suggested [68].

Before presenting the n-dimensional spiral optimization algorithm, it is worth understanding the two dimensional optimization model as a basis for the n-dimensional version of the algorithm.

### III.1. Two-dimensional spiral optimization

Rotating a point in a 2-dimensional orthogonal coordinate system (as shown in Fig. 6) in the counter-clockwise direction around the origin by $\theta$ can be expressed as:

$$ x' = R(\theta) x $$  \hspace{1cm} (1)

Where

$$ R_2(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} $$  \hspace{1cm} (2)

Hence, the two dimensional algorithm moves from one point to another as:

$$ \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = rR_2(\theta) \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} $$  \hspace{1cm} (3)

Where $\theta$ is the rotation angle around the origin ($0 \leq \theta \leq 2\pi$) and $r$ is the convergence rate of distance between a point and the origin at each $k$ ($0 < r < 1$).

The spiral model presented earlier has a center only at the origin. Hence, it should be extended to have center at an arbitrary point $x^*$ as:

$$ x(k+1) = rR_2(\theta)x(k) - (rR_2(\theta) - I_2)x^* $$  \hspace{1cm} (4)

![Figure 5. Logarithmic spiral](image)

![Figure 6. Rotation in $x_1$-$x_2$ plane](image)
This suggests the following optimization algorithm:

- **Preparation**: select the number of search points \( m > 2 \), the parameters \( \theta \) and \( r \) and the maximum number of iterations \( k_{max} \).

- **Initialization**: initialize randomly the points; \( x^*_i(0) i = 1..m \); in the feasible region and the center \( x^* \) as the point with the least fitness value.

- **Updating \( x_i \)**:

  \[
  x_i(k + 1) = r R_\theta(x_i(k)) - (r R_\theta(\theta) - I_\theta) x^*_i \quad \text{for} \quad i = 1..m. 
  \]

- **Updating \( x^* \)**: Select \( x^* \) as the point with the least fitness function in the updated set of points.

- **Check for termination criterion**: If \( k = k_{max} \), then stop. Otherwise, start a new iteration.

### III.2. n-dimensional spiral optimization

The extension of the two-dimensional optimization algorithm presented earlier is straightforward as one must understand how rotation in an \( n \)-dimensional space is done. Rotation in \( n \)-dimension is performed in the same way as the two-dimensional rotation taking two dimensions at a time. This is defined for dimensions \( i,j \) as:

\[
R_{i,j} = \begin{bmatrix}
1 & 0 & i & 0 & 0 & j & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Hence, there are \( \frac{n(n-1)}{2} \) rotation matrices. The resulting rotation matrix is then [55]:

\[
R_n(\theta) = \prod_{i=1}^{n-1} \left( \prod_{j=i}^{n} R_{i,j-1,i,j}(\theta) \right) 
\]

Hence the \( n \)-dimensional algorithm may be formulated similar to the two-dimensional algorithm as:

- **Preparation**: select the number of search points \( m > 2 \), the parameters \( \theta \) and \( r \) and the maximum number of iterations \( k_{max} \).

- **Initialization**: initialize randomly the points; \( x_i(0) i = 1..m \); in the feasible region and the center \( x^* \) as the point with the least fitness value.

- **Updating \( x_i \)**:

  \[
  x_i(k + 1) = r R_\theta(x_i(k)) x_i(k) - (r R_\theta(\theta) - I_\theta) x^*_i \quad \text{for} \quad i = 1..m. 
  \]

- **Updating \( x^* \)**: Select \( x^* \) as the point with the least fitness function in the updated set of points.

### IV. Results and discussions

#### IV.1. Mathematical formulation

This paper deals with harmonic elimination/reduction in Inverters using Pulse Width Modulation by solving a system of nonlinear equations. Equations are used to determine switching angles of an Inverter. Switching angles play an important role to produce the desired output by eliminating selected harmonics.

To eliminate the specific \((N-1)\) odd harmonics, we must obtain the values of \( N \) switching angles, namely \( \alpha_1, \alpha_2, ..., \alpha_N \) using Equation (9) and Equation (10) under the constraint given in Equation (11)

\[
V_{out}(t) = \sum_{n=1}^{N} V_n \sin(n\omega t)\tag{9}
\]

\[
V_n = \frac{4V_{dc}}{n\pi} \sum_{k=1}^{N} \cos(n\alpha_k), \quad n = 1,3,5 \tag{10}
\]

\[
0<\alpha_1<\alpha_2<...<\alpha_N<\frac{\pi}{2} \tag{11}
\]

Where:

- \( N \) is the number of switching angles per quarter.
- \( V_{dc} \) is the amplitude of DC source.
- \( n \) is the odd harmonic order.
- \( \alpha_k \) is the \( k^{th} \) switching angle.

The equation derived for Total Harmonic Distortion factor of the output voltage of an inverter is used in order to reduce the harmonics that are produced in the inverter. The switching angles which are required for the THD are calculated to minimize the fitness function by the spiral optimization technique is the percentage of the Total Harmonic Distortion is given by the formula:

\[
THD = \sqrt{\frac{\sum_{i=1}^{p} \cos(\alpha_i)^2}{\sum_{i=1}^{p} \frac{\cos(\alpha_i)(\cos(\alpha_i) + 2)}{\cos(\alpha_i)^2 + 2}} - \left(\frac{\cos(\alpha_i)(\cos(\alpha_i) + 2)}{\cos(\alpha_i)^2 + 2}\right)^2} \tag{12}
\]

#### IV.2. Results and Discussions

The simulation of the Selective Harmonic Elimination technique (SHE) for 3-Phase Voltage Source Inverter (VSI) is presented in this section. The optimum firing angles are calculated using Spiral Optimization Technique.

MATLAB/SIMULINK® is used to analyze the results of the SHE simulation in PWM inverters with different numbers of switching angles.

A. SHE in three phase two level PWM VSI with three angles:

In the SIMULINK scheme of Fig. 7, two models are simulated simultaneously. The first model (the upper side) represents the Selective Harmonic Elimination PWM technique while the second (at
the bottom) is the standard sinusoidal PWM (SPWM) technique. The results from both models are studied and compared. The work starts with the elimination of two harmonics, which means that three firing angles are required (the third is used to control the modulation index ‘\( m \)’). The first angle (\( \alpha_1 \)) is used to control the modulation index, while the remaining two angles (\( \alpha_2 \) and \( \alpha_3 \)) are used to eliminate two preselected low order harmonics (the 5th and the 7th).

The line to line voltages and currents of both the SHEPWM and SPWM models are illustrated in Fig. 8. The first and second waveforms are the line to line voltage and currents of the SHE-PWM module, while the third and the fourth are the ones of the SPWM module. One can remark a small difference in the shapes of the SPWM and SHEPWM modules. The ones of the SHEPWM module is sharper and could be considered closer to a sinusoidal shape than the ones of the SPWM module. This is due to the elimination of only two low order harmonics. Better results and cleaner waveforms are expected when eliminating more low order harmonics.

Table 2 gives the THD corresponding to the different modulation index values in the line to line voltages of both SPWM and SHEPWM. It is to be noted that the THD parameter gets reduced with the increase of the modulation index in both SPWM and SHEPWM models.

Table 3 shows the resulting undesirable harmonic levels where a significant reduction in the unwanted harmonics is noticed (less than 2%).
Table 3. 5th and 7th harmonics percentage compared to the fundamental

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>SHEPWM</th>
<th>SPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65</td>
<td>1.2%</td>
<td>68.12%</td>
</tr>
<tr>
<td>0.85</td>
<td>1.13%</td>
<td>50.4%</td>
</tr>
<tr>
<td>1.00</td>
<td>0.79%</td>
<td></td>
</tr>
</tbody>
</table>

B. SHE in Three-Phase Two-Level PWM VSI With Five Angles

In order to eliminate four pre-selected low order harmonics (5th, 7th, 11th, and 13th harmonics); a change is made in the preloaded function of the SHEPWM block. The results are shown in Table 4. The resulting waveforms are displayed in Fig. 9. The first and second graphs represent the line to line voltage and current waveforms of the SHEPWM module, while the third and fourth graphs are the line to line current and voltage waveforms of the SPWM module. A remarkable difference between the shape of the SHEPWM and the one of the SPWM voltage and current graphs can be noticed. The SHEPWM looks sharper and much closer to a sinusoidal shape than the ones of the SPWM module; which proves the effectiveness of the selective harmonic elimination technique.

Table 4. The optimum solutions using Spiral Optimization Technique

<table>
<thead>
<tr>
<th>Modulation index</th>
<th>(\alpha_1) (deg)</th>
<th>(\alpha_2) (deg)</th>
<th>(\alpha_3) (deg)</th>
<th>(\alpha_4) (deg)</th>
<th>(\alpha_5) (deg)</th>
</tr>
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<tbody>
<tr>
<td>0.1</td>
<td>19.12</td>
<td>20.45</td>
<td>39.08</td>
<td>40.72</td>
<td>59.12</td>
</tr>
<tr>
<td>0.2</td>
<td>18.23</td>
<td>20.90</td>
<td>38.16</td>
<td>41.44</td>
<td>58.25</td>
</tr>
<tr>
<td>0.3</td>
<td>17.32</td>
<td>21.30</td>
<td>37.21</td>
<td>42.16</td>
<td>57.35</td>
</tr>
<tr>
<td>0.4</td>
<td>16.41</td>
<td>21.78</td>
<td>36.24</td>
<td>42.88</td>
<td>56.45</td>
</tr>
<tr>
<td>0.5</td>
<td>15.47</td>
<td>22.19</td>
<td>35.24</td>
<td>43.59</td>
<td>55.52</td>
</tr>
<tr>
<td>0.6</td>
<td>14.52</td>
<td>22.58</td>
<td>34.20</td>
<td>44.29</td>
<td>54.57</td>
</tr>
<tr>
<td>0.7</td>
<td>13.54</td>
<td>22.91</td>
<td>33.10</td>
<td>44.96</td>
<td>53.58</td>
</tr>
<tr>
<td>0.8</td>
<td>12.53</td>
<td>23.17</td>
<td>31.92</td>
<td>45.59</td>
<td>52.53</td>
</tr>
<tr>
<td>0.9</td>
<td>11.48</td>
<td>23.30</td>
<td>30.61</td>
<td>46.13</td>
<td>52.37</td>
</tr>
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<td>1.0</td>
<td>10.36</td>
<td>23.19</td>
<td>29.07</td>
<td>46.43</td>
<td>49.94</td>
</tr>
</tbody>
</table>

The results show a significant reduction of the THD percentage when increasing the values of the modulation index. It is also remarkable that the values of THD in SHEPWM module are always lower than the ones of the SPWM module which is an indicator of the effectiveness of the SHE technique in reducing harmonics in the output waveform of the PWM VSI.

Further details are shown in Table 5, where the percentages of the amplitude of the undesired harmonics compared to the fundamental are displayed. The percentage of the amplitude of the preselected harmonics is always less than 1.8%; this indicates the success of the elimination or at least reduction of the undesired harmonics from the output waveform of the voltage source inverter.

Table 5. THD parameter of the SPWM and SHEPWM modules with 5 firing angles

<table>
<thead>
<tr>
<th>Modulation Index</th>
<th>SPWM</th>
<th>SHEPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>12.13%</td>
<td>10.48%</td>
</tr>
<tr>
<td>0.7</td>
<td>10.55%</td>
<td>9.80%</td>
</tr>
<tr>
<td>0.8</td>
<td>9.21%</td>
<td>8.67%</td>
</tr>
<tr>
<td>0.9</td>
<td>8.06%</td>
<td>7.29%</td>
</tr>
<tr>
<td>1.0</td>
<td>6.92%</td>
<td>6.06%</td>
</tr>
</tbody>
</table>

C. SHE in Three-Phase Two-Level PWM VSI with Seven Angles

The task here concerns the elimination of seven preselected low order harmonics (5th, 7th, 11th, 13th, 17th, and 19th harmonics). The obtained results are shown in Table 7. Fig. 10 represents the resulting line to line currents and voltages waveforms of the SHEPWM and SMPWM modules with seven firing angles (6 eliminated harmonics). The difference is remarkable between the shapes of the current and...
voltage waveforms of the SHEPWM and SPWM blocks, the first one looks sharper and more like a sinusoidal waveform than the second one.

Table 6. Undesired harmonic percentages relative to the fundamental

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>SHEPWM</th>
<th>SPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th</td>
<td>0.54%</td>
<td>0.35%</td>
</tr>
<tr>
<td>7th</td>
<td>1.72%</td>
<td>0.22%</td>
</tr>
<tr>
<td>11th</td>
<td>0.26%</td>
<td>0.28%</td>
</tr>
<tr>
<td>13th</td>
<td>0.21%</td>
<td>0.29%</td>
</tr>
</tbody>
</table>

Table 8 presents the THD parameter corresponding to the different modulation index values in the line to line voltages of both the SPWM and SHEPWM models.

Table 7. The optimum solutions using Spiral Optimization Technique

<table>
<thead>
<tr>
<th>Modulation index</th>
<th>(\alpha_1)</th>
<th>(\alpha_2)</th>
<th>(\alpha_3)</th>
<th>(\alpha_4)</th>
<th>(\alpha_5)</th>
<th>(\alpha_6)</th>
<th>(\alpha_7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.59</td>
<td>15.32</td>
<td>29.35</td>
<td>30.48</td>
<td>44.33</td>
<td>60.65</td>
<td>74.40</td>
</tr>
<tr>
<td>0.2</td>
<td>1.19</td>
<td>15.66</td>
<td>28.70</td>
<td>30.96</td>
<td>43.65</td>
<td>61.30</td>
<td>73.81</td>
</tr>
<tr>
<td>0.3</td>
<td>1.77</td>
<td>15.99</td>
<td>28.04</td>
<td>31.43</td>
<td>42.97</td>
<td>61.96</td>
<td>73.23</td>
</tr>
<tr>
<td>0.4</td>
<td>2.36</td>
<td>16.31</td>
<td>27.36</td>
<td>31.89</td>
<td>42.26</td>
<td>62.63</td>
<td>72.62</td>
</tr>
<tr>
<td>0.5</td>
<td>2.93</td>
<td>16.63</td>
<td>26.67</td>
<td>32.34</td>
<td>41.54</td>
<td>63.31</td>
<td>72.07</td>
</tr>
<tr>
<td>0.6</td>
<td>3.51</td>
<td>16.92</td>
<td>25.95</td>
<td>32.76</td>
<td>40.79</td>
<td>64.00</td>
<td>71.50</td>
</tr>
<tr>
<td>0.7</td>
<td>4.07</td>
<td>17.12</td>
<td>25.19</td>
<td>33.14</td>
<td>40.00</td>
<td>64.71</td>
<td>70.95</td>
</tr>
<tr>
<td>0.8</td>
<td>4.62</td>
<td>17.39</td>
<td>24.39</td>
<td>33.46</td>
<td>39.15</td>
<td>65.45</td>
<td>70.42</td>
</tr>
<tr>
<td>0.9</td>
<td>5.16</td>
<td>17.51</td>
<td>23.49</td>
<td>33.67</td>
<td>38.18</td>
<td>66.26</td>
<td>69.95</td>
</tr>
<tr>
<td>1.0</td>
<td>5.68</td>
<td>17.46</td>
<td>22.45</td>
<td>33.63</td>
<td>36.99</td>
<td>67.22</td>
<td>69.62</td>
</tr>
</tbody>
</table>

A similar behavior regarding the THD is noticed compared to the results in the three and five angles sections. It is remarkable that the THD of the SHEPWM technique is lower than the THD of the SPWM technique.

Table 8. THD parameter of the SPWM and SHEPWM modules with 7 firing angles

<table>
<thead>
<tr>
<th>Modulation Index</th>
<th>SPWM</th>
<th>SHEPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>12.13%</td>
<td>8.75%</td>
</tr>
<tr>
<td>0.7</td>
<td>10.55%</td>
<td>7.21%</td>
</tr>
<tr>
<td>0.8</td>
<td>9.21%</td>
<td>6.56%</td>
</tr>
<tr>
<td>0.9</td>
<td>8.06%</td>
<td>6.14%</td>
</tr>
<tr>
<td>1.0</td>
<td>6.92%</td>
<td>5.41%</td>
</tr>
</tbody>
</table>

Figure 10. Output voltage and current waveforms of SHEPWM and SMWP module

More details can be seen in Table 9, where the percentages of amplitudes of the undesired harmonic compared to the amplitude of the fundamental harmonic in both SPWM and SHEPWM (at different modulation index values) are displayed.

Table 9. Undesirable harmonic percentages relative to the fundamental

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>SHEPWM</th>
<th>SPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th</td>
<td>0.70%</td>
<td>0.71%</td>
</tr>
<tr>
<td>7th</td>
<td>1.22%</td>
<td>1.34%</td>
</tr>
<tr>
<td>11th</td>
<td>0.32%</td>
<td>0.81%</td>
</tr>
<tr>
<td>13th</td>
<td>0.08%</td>
<td>0.32%</td>
</tr>
<tr>
<td>17th</td>
<td>1.88%</td>
<td>1.02%</td>
</tr>
<tr>
<td>19th</td>
<td>1.40%</td>
<td>0.97%</td>
</tr>
</tbody>
</table>
V. Conclusion

In this paper, harmonic reduction and elimination based on the spiral optimization technique has been addressed. The switching angles of the inverter are sought such as to minimize the THD and/or completely eliminate some selected harmonics. The simulation results prove the validity of the analysis and the feasibility of the utilized technique compared to the conventional ones. The obtained results are promising as the values of the harmonic distortion factor are of practical use in the modern power electronic applications such as the grid-connected renewable energy sources.

VI. References


Please cite this Article as: